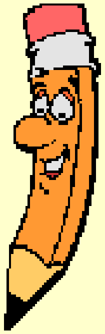

Writing and Scoring Open-Ended Question in Math

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Closed Ended Questions

Many of the questions we traditionally ask students call for a single number, figure, or mathematical object.

Examples:

What are the prime numbers between 10 and 20?

Which triangles in a set of triangles are congruent?

Factor $3x^2 - 20x - 7$

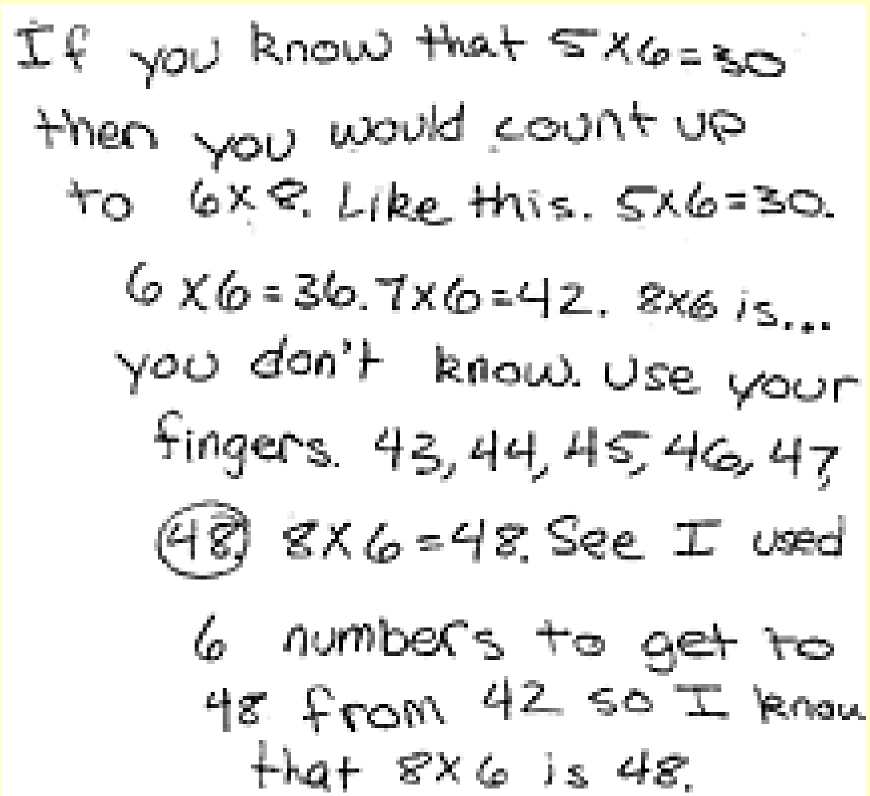
These are *closed ended* because the expected answers are predetermined and specific.

Open Ended Questions

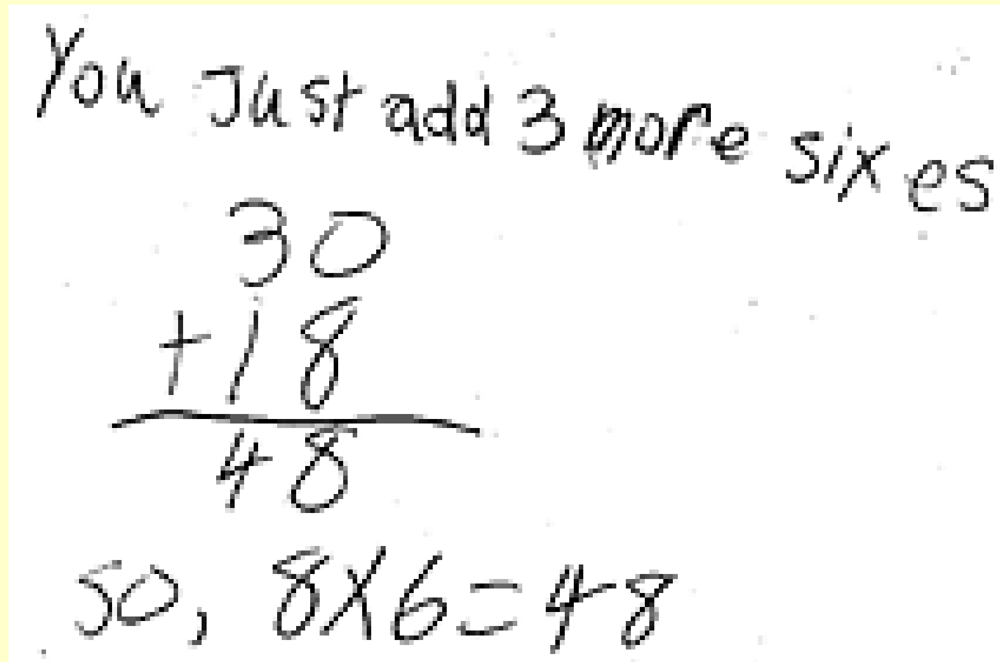
In contrast, open-ended questions allow a variety of correct responses and elicit a different kind of student thinking.

1. Suppose you forgot what 8 x 6 is, but you remembered that 5 x 6 is 30. How could you use this fact to figure out what 8 x 6 is?

Responses



If you know that 5x6=30 then you would count up to 6x8. Like this. 5x6=30. 6x6=36. 7x6=42. 8x6 is... you don't know. Use your fingers. 43, 44, 45, 46, 47, 48. 8x6=48. See I used 6 numbers to get to 48 from 42 so I know that 8x6 is 48.



You just add 3 more sixes

$$\begin{array}{r} 30 \\ +18 \\ \hline 48 \end{array}$$

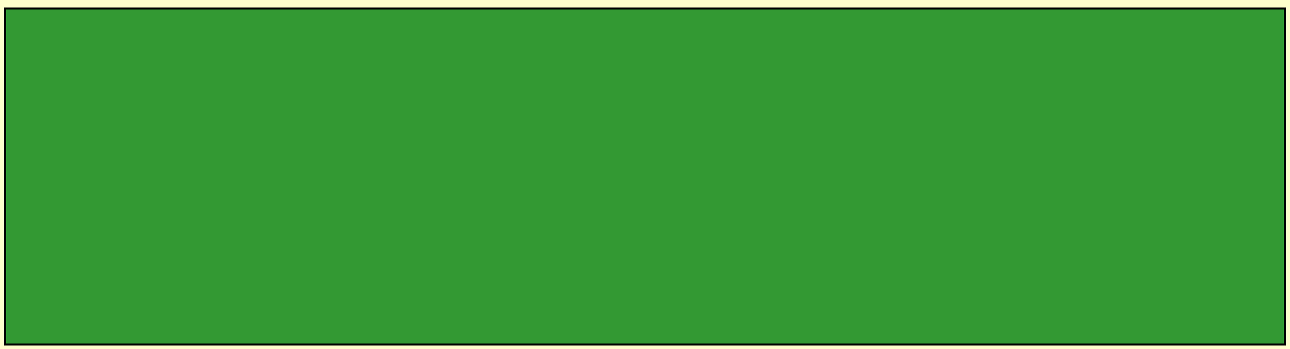
so, 8X6=48

Both responses demonstrate the ability to decompose the original multiplication problem into subproblems.

The open-ended nature of the question allows students to demonstrate their own ways of solving the problem.

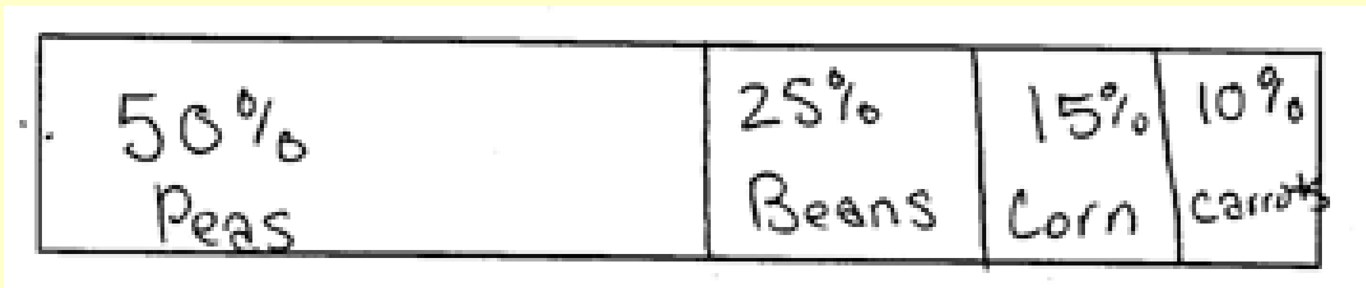
Open Ended Questions

2. Divide and label the garden plot below so that 50% of the garden is planted in peas, 25% is planted in beans, 15% is planted in corn, and 10% is planted in carrots.



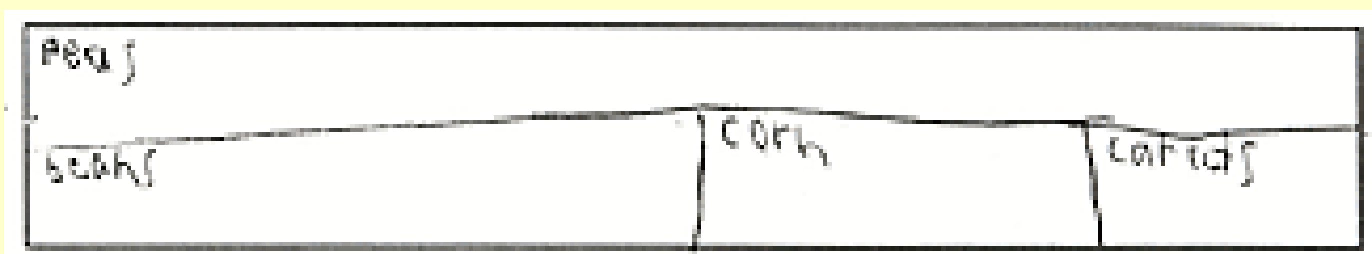
Responses

Response 1:



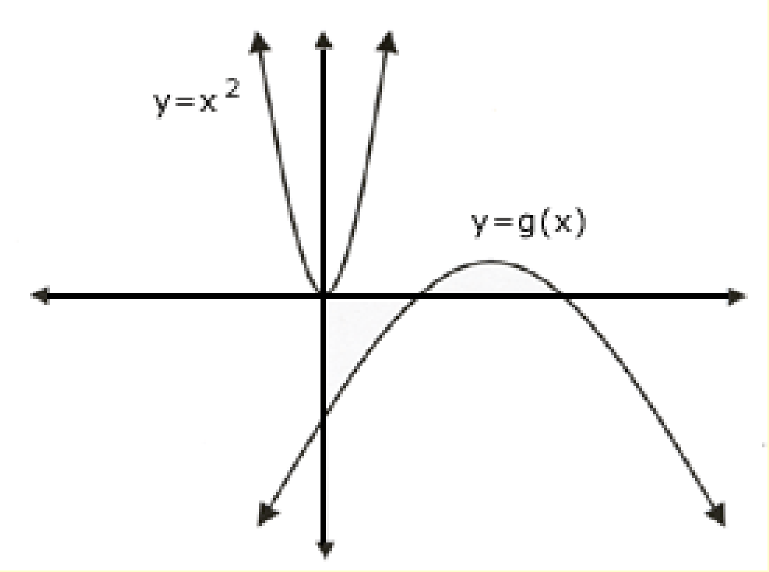
Although both responses are correct, each student made a different decision about how to subdivide the rectangle.

Response 2:

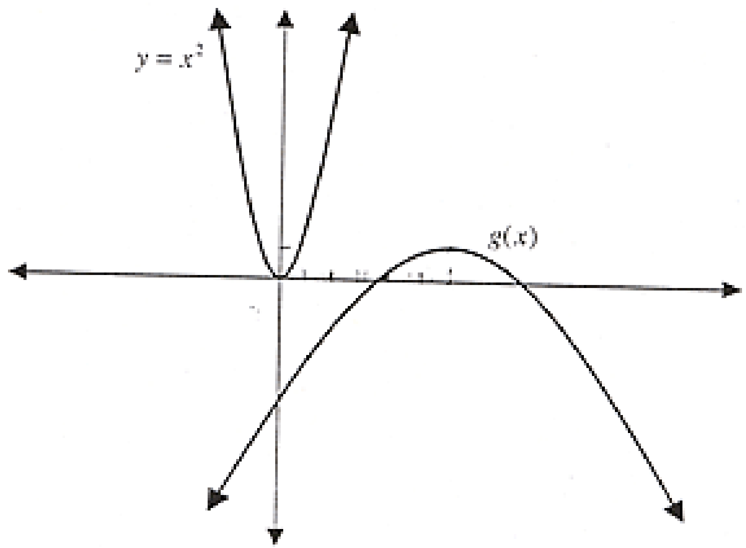


Open Ended Questions

3. Give a reasonable equation for *g(x)* in the form $g(x) = a(x - h)^2 + k$. Explain why your equation is reasonable.



Response



$g(x) = -.5(x-4)^2 + 1$

This is a reasonable equation. The (-) flips the graph so it opens downwards, the (.5) widens the graph, the (x-4) shifts the graph 4 units to the right, and the (+1) shifts the graph one unit up. The final product strongly resembles the graph of g(x). I picked these numbers in relation to the given graph of $y = x^2$. The changes made to this standard formula will make a graph strongly resembling the graph of g(x).

Expected Response

There are many correct responses to this question.

The condition that must be true are:

$-1 < a < 0$, $h > 0$, $k > 0$, and h is approximately four times k (assuming the scaling on the x and y axes is the same).

Closed Ended/Open Ended Questions

1. Both closed-ended and open-ended questions are appropriate for assessing students' mathematical thinking.

Closed Ended/Open Ended Questions

2. A test consisting solely of open-ended questions would take an inordinate amount of time to grade and might not cover the curriculum adequately.

Closed Ended/Open Ended Questions

3. Closed-ended questions are a reasonable way to sample students' understanding of a broad range of topics.

   But closed-ended questions do not allow students to reveal their thinking processes as well as open-ended questions.

Characteristics of a Good Open-Ended Question

1. Should Involve significant mathematics.

Assessment items often signal students as to what is valued and what is important. Therefore, it is critical that open-ended mathematics assessment items involve significant mathematics.

Open-ended items can often have several objectives. This gives students the opportunity to demonstrate their understanding of connections across mathematical topics and how mathematics can model real world phenomena.

Characteristics of a Good Open-Ended Question

2. Should Elicit a Range of Responses.

Items that require students to explain their thinking are more likely to encourage a wide range of responses because not all students think alike.

Can an equilateral triangle have a right angle? Why or why not?

How could students respond to this question?

Is there just one response to this question?

What ideas could they compile?

Characteristics of a Good Open-Ended Question

3. <u>Should Require Communication</u>.

By design, open-ended questions give students opportunities to communicate their thinking.

Mary claims that you can find the area of any 30-60-90 triangle given the length of only one side. Is Mary correct or not? Justify your answer.

Responses

Mary is right. If you know one side you can either divide by $\sqrt{3}$ *or 2 or multiply by 2 or* $\sqrt{3}$*. Then you can just multiply the height and the base, divide by 2, and you got it.*

Mary goofed. The angles are all different so the side lengths are all different. Knowing just one side is a start but you have to have two sides (base and height) to get the area.

The first student sees the relevance of the relationship among the sides of a 30-60-90 triangle, whereas the second student, who may be aware of this relationship, does not see its relevance in the context of this problem.

Advantage of Open-Ended Questions

When students are required to **communicate** their **reasoning processes**, we have a better chance of understanding what they know and can apply it to a given problem.

Characteristics of a Good Open-Ended Question

4. Should be Clearly Stated.

The question should have a **clear purpose** even though there might be many possible responses.

Students should **know what is expected** of them and **what the teacher expects** as a good and complete response.

Sharing a variety of responses with students and asking them to evaluate the responses helps them determine what constitutes a good response.

It is important to help them **develop** their communication skills and their ability to **analyze** how well their writing communicates their reasoning.

Process Standard

Characteristics of a Good Open-Ended Question

5. Should Lend Itself to a Scoring Rubric.

Every assessment item lends itself to at least a two-point scoring rubric: **right or wrong**

But the purpose of open-ended questions is to provide students with the opportunity to **communicate** their understanding in something other than a right vs. wrong scenario.

Characteristics of a Good Open-Ended Question

In designing a question you must decide whether it is possible to conceive of responses that have some value (better than a score of 0) but are not worthy of full credit (less than a score of 3).

Giving students partial credit is a familiar notion, and using a rubric formalizes the process to help ensure fairness.

Characteristics of a Good Open-Ended Question

One criterion for a good open-ended question is that it will elicit responses that are amenable to partial credit according to some established rubric.

Can an equilateral triangle have a right angle?

This question does not allow an assessment that involves partial credit.

Can an equilateral triangle have a right angle?
Why or why not?

Allows for a variety of responses.

Creating an Open-Ended Question

One way to create new items is to **change closed-ended questions into open-ended ones**.

The questions will become more conceptually oriented and require students to communicate their thinking processes.

Find the LCM of 18 and 24.	Why can't 48 be the LCM of 18 and 24?

Some Suggestions

Ask Students to **Create a Situation** or **an Example** That Satisfies Certain Conditions

Questions of this type require students to recognize the defining characteristics of the underlying concept. Students must take what they know about a concept and apply it to create an example.

Fill in values for *a* and *b* to make the equation below true. Explain why your equation is true.

$$\sqrt{a} = 2\sqrt{b}$$

Some Suggestions

Ask Students to **Create a Situation** or **an Example** That Satisfies Certain Conditions

Questions of this type require students to recognize the defining characteristics of the underlying concept. Students must take what they know about a concept and apply it to create an example.

Draw a quadrilateral ABCD that has one and only one line of symmetry. Explain why your quadrilateral satisfies the given condition.

Write an irrational number whose square is smaller than itself. Explain why your number fits the criteria or argue that it is not possible to write such a number.

Write a data set consisting of 10 numbers so that the range is twice the median. Show that your data set satisfies the criteria.

Give the dimensions of a cone and a cylinder that have the same volume. Show that the two solids have the same volume.

Write an equation of a circle that contains the points (-4, -3) and (6,1). Graph your circle and explain why its equation satisfies the given condition.

Ask Students to **Explain Who Is Correct and Why**

These types of items present two or more views of some mathematical concept or principle and the student has to decide which of the positions is correct and why.

The following responses were given when students were asked to evaluate 2^8 :

Michael: $2^8 = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16$

Damon: $2^8 = 2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2 = 256$

Dawn: $2^8 = 2^2 \cdot 2^2 \cdot 2^2 = 64$

Which student is correct? Explain why that student is correct.

Perry claims that 3 is not a zero of the polynomial below. Janice claims 3 could be a zero of the polynomial, depending on the value of *a*. Who is correct and why?

$$2x^4 + ax^3 + 3x^2 - 5x + 10$$

Kent calculated tan Ø and sin Ø (for a particular angle Ø) and claimed that tan Ø < sin Ø. Wally said this was impossible. Who is correct and why?

Melanie claims that there is some value for a for which the system of linear equations below has no solution:

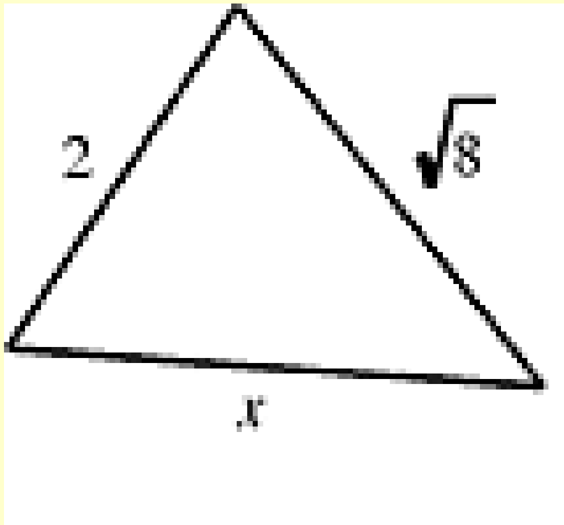
$$2x - 5y = 8$$

$$3x - 6y = a$$

Jeffrey disagrees and claims that there will be a solution for the system regardless of the value of a. Who is correct and why?

Ask Students to **Solve or Explain the Problem/Solution in Two or More Ways**

Give two different whole number values for *x* such that it would be possible to construct a triangle with the given lengths. Explain why your values for *x* will allow you to make a triangle.



Using different combinations from the list of number systems, write three true statements of the form *All* __________ *are* _________________.

Complex numbers

Integers

Irrational numbers

Natural numbers

Rational numbers

Real numbers

Whole numbers

Explain why your statements are true.

Using different combinations from the list of polygons, write three true statements of the form *All* ___________ *are* ____________________.

Kites	**Parallelograms**	**Quadrilaterals**
Rectangles	**Rhombi**	**Squares**
	Trapezoids	

Explain why your statements are true.

When you create open-ended items, **make sure they are really different from traditional items**.

For example, the following item is really no different than simply asking students to solve the equation:

Johnny said the solution to the inequality $(x - 2)(x - 4) < 0$ is $2 < x < 4$.
Susie solved the inequality and got $x < 2$ or $x > 4$.
Who is correct and why?

Johnny was given the inequality $(x - 2)(x - 4) < 0$. He said that $x = 3$ is the only solution to this inequality. Explain to Johnny why his statement about the inequality is incorrect.

The question should require students to explain their reasoning, not simply to reproduce an algorithm.

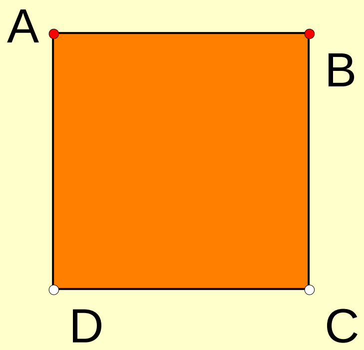
- Mr. Stephens went to the grocery story to buy cereal for his children. There are 6 different kinds of cereal his children like, and Mr. Stephens want to buy 2 boxes. He reasons that he has 6 choices for the first box and 5 choices for the second box and hence he has 6 x 5 =30 different pairs of cereal brands he could buy. Is Mr. Stephen’s reasoning correct? Why or why not?

- List and graph a set of 10 data points for which the line of best fit has a negative slope. Explain why your data would have such a line of best fit.

- Give values for a, b, and c so that the expression below can be simplified. Explain why your expression can be simplified.

$$\frac{x^a + x^b}{x^c}, x \neq 0$$

- Draw a square that has twice the area of square ABCD. Explain why your square has twice the area.



- Why is the surface area of a cube measured in square units and the volume of a cube measured in cubic units?

Using a Rubric to Score Student Responses

A rubric is a name for what teachers have been doing for a long time.

Sometimes a very simple rubric is used:

right or wrong, full credit or no credit.

Sometimes rubrics are more complex:

right, wrong, or sort of.

We think to ourselves, *How much did I count off for that?*

We look back and then say *minus 2 for sign error* to keep our grading consistent.

What is a Rubric

- A rubric is a list of indicators that helps us rank responses based on some criteria.
- A rubric can be analytic or holistic.

Types of Rubrics

- An **analytic rubric** is divided into several dimensions.
  - The dimensions might include communication, mathematical correctness, and completeness.
- **Holistic rubrics** help teachers assess the whole task on one scale.
  - For open-ended questions, a holistic rubric is usually the most effective and also user friendly.

Reasons for a Rubric

1. Rubrics helps us **focus on what students know and can do** rather than on what they do not know and cannot do.

Question:

Write two mixed numbers whose sum is

$$3\frac{1}{2}$$

and explain how they know that their two numbers satisfy the condition.

One student provides **two mixed numbers** whose sum is not $3\frac{1}{2}$

and another student provides **two improper fractions** whose sum is not $3\frac{1}{2}$.

Both students are incorrect, BUT the first student knows something the second student does not.

By providing mixed numbers, the first student shows that he knows what mixed numbers are.

A rubric can help focus your attention on what **mathematical knowledge is apparent** from the response.

Reasons for a Rubric

2. A rubric helps us keep **grading consistent.**

Question:

Why is multiplication the appropriate operation to solve a particular problem.

Student A responds, **"To get the right answer."**

Student B responds, **"If you don't multiply, it is going to be wrong."**

Student C responds, **"It is what you have to do in order to get the answer right."**

The teacher who graded these responses scored student A's response with 0 points, student B's response with 2 points and student C's response with 3 points (scale: 0-3).

The answers are essentially the same. A rubric can help prevent these kinds of scoring inconsistencies.

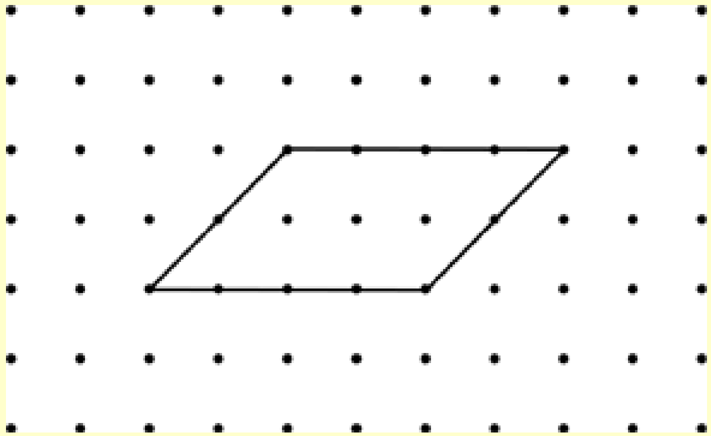
Reasons for a Rubric

3. Students **can evaluate previous students' responses** to a question using the same scoring rubric we will use.

This helps students **better understand our expectations** and **learn what differentiates** high-level responses from low-level responses.

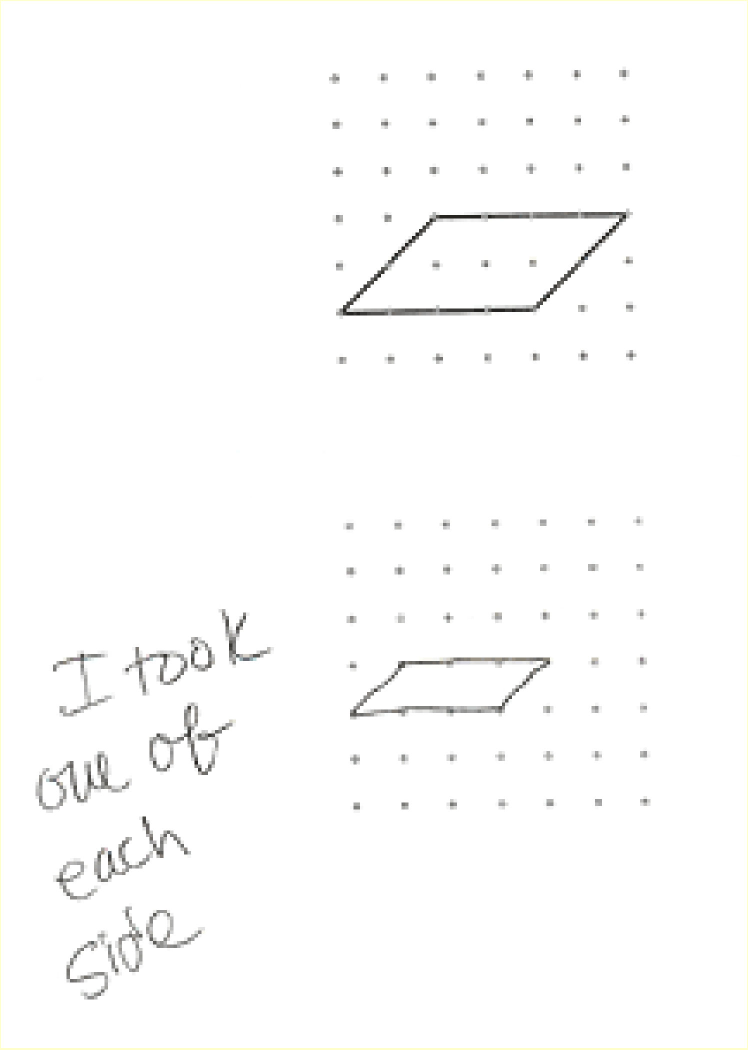
Assessment thus becomes less mysterious.

0	Response indicates no appropriate mathematical reasoning
1	Response indicates some mathematical reasoning but fails to address the item's main mathematical ideas
2	Response indicates substantial and appropriate mathematical reasoning but is lacking in some minor way(s)
3	Response is correct and the underlying reasoning process is appropriate and clearly communicated



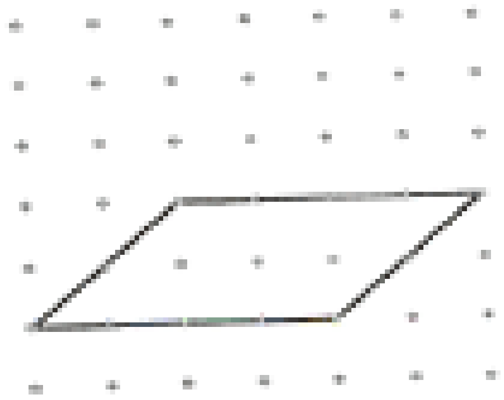
Draw a parallelogram on the dot paper below that is similar but not congruent to the parallelogram above. Explain how you know the two parallelograms are similar but not congruent.

Sample 1

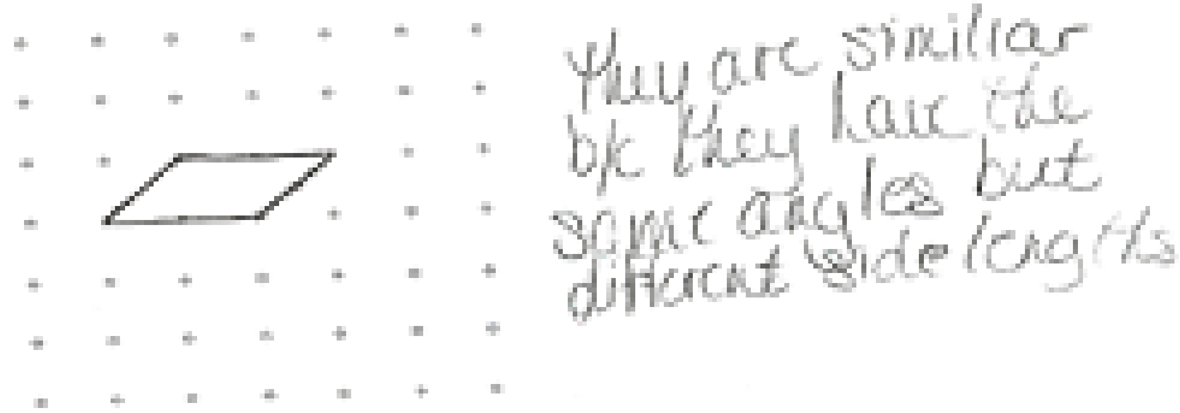


I took one of each side

Sample 2



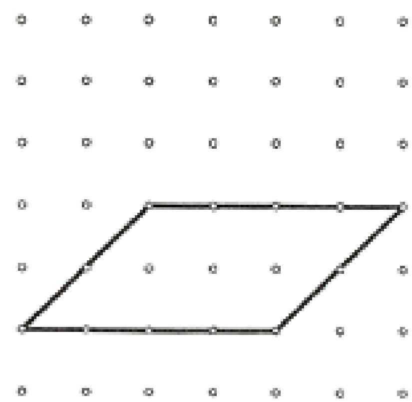
They are similar b/c they have the same angles but different side lengths

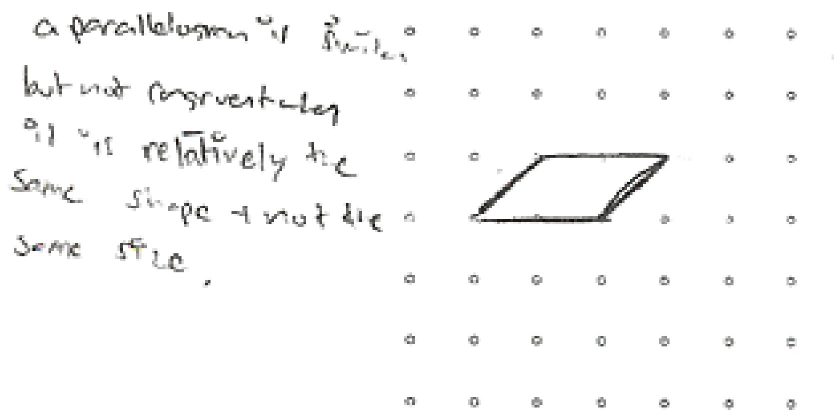


Sample 3

A parallelogram is similar but not congruent

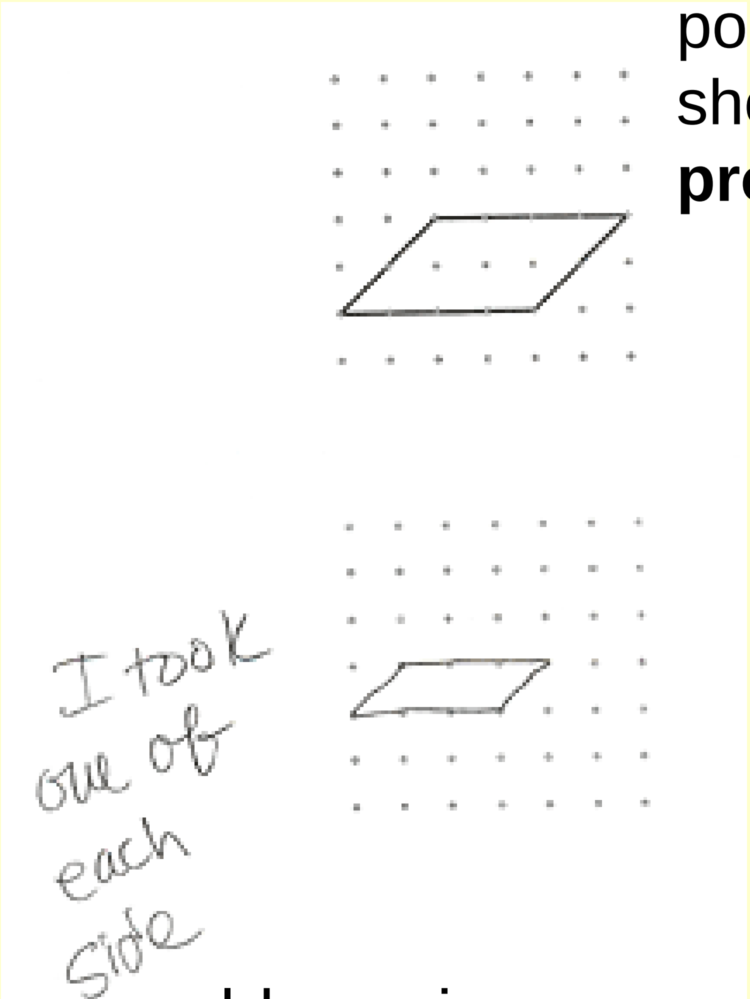
It is relatively the same shape and not the same size.





Sample 1

This student showed some **evidence of appropriate mathematical reasoning in that (s)he drew a parallelogram**.

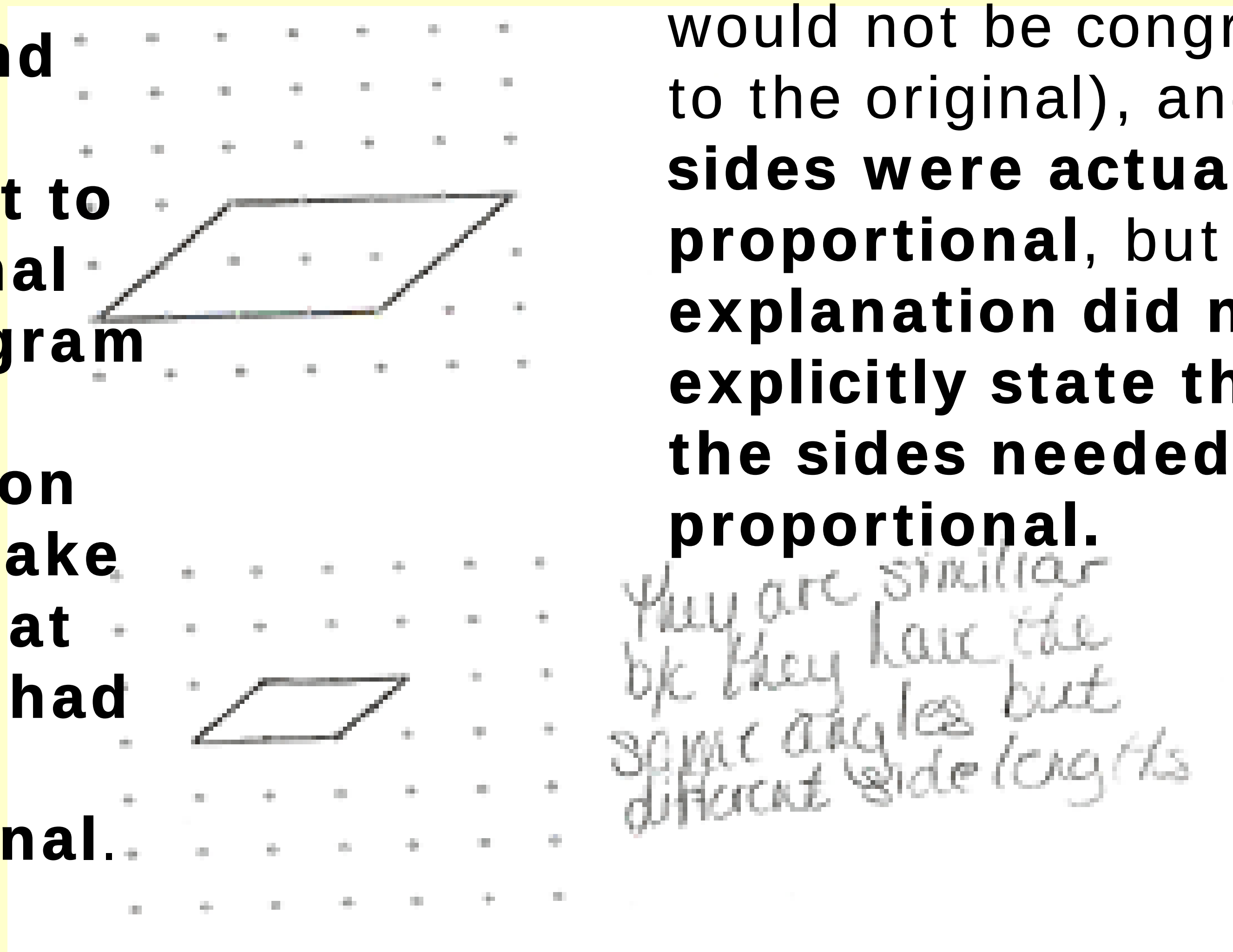


However, (s)he **missed the item's main** mathematical point that the sides should be **proportional**.

This response would receive a **score of 1.**

Sample 2

This student **drew a parallelogram that was similar and not congruent to the original parallelogram**, but the **explanation did not make it clear that the sides had to be proportional**.



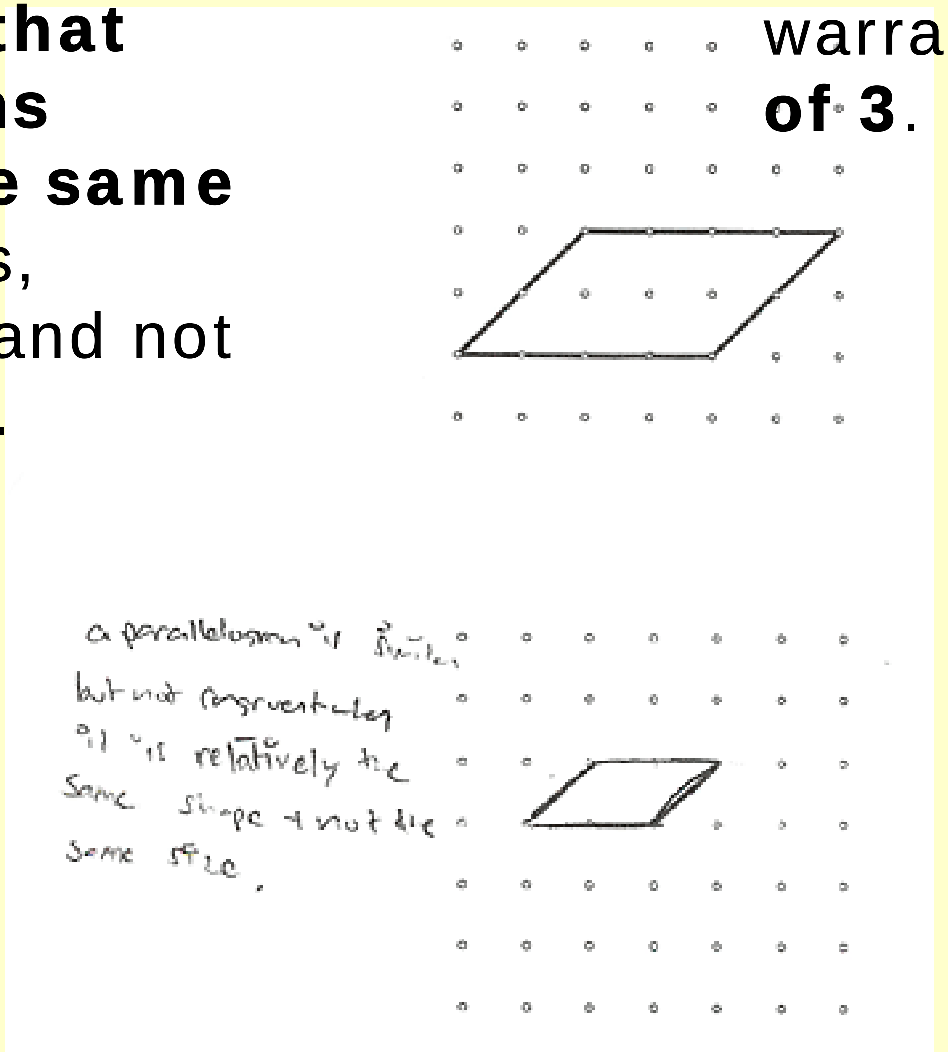
(S)he pointed out that the sides should have different lengths (so that the new parallelogram would not be congruent to the original), and **her sides were actually proportional**, but **her explanation did not explicitly state that the sides needed to be proportional.**

This response was **scored a 2**.

Sample 3

This student **drew a parallelogram that was similar but not congruent and pointed out that similar means relatively the same shape** (that is, proportional) and not the same size.

The word "relatively" makes the difference, warranting a **score of 3**.



A score of 0 would be given to a response that is **a shape other than a parallelogram** or a parallelogram that is not similar to the original shape, because there would be no evidence of appropriate mathematical reasoning.

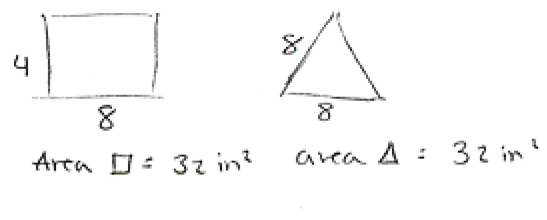
- As with all grading, different teachers will grade open-ended items differently, because a particular teacher focuses on some aspect of the response more than another teacher does.
- It is less important that all teachers use the rubric in the same way and more important that a particular teacher uses it the same way for all of her or his students.
- Once your students become familiar with the rubric and how you use it, they will learn what kind of information they need to include in their responses in order to earn full credit.

- Talking with your colleagues about student responses to open-ended questions and how to score the responses using a rubric is very helpful.

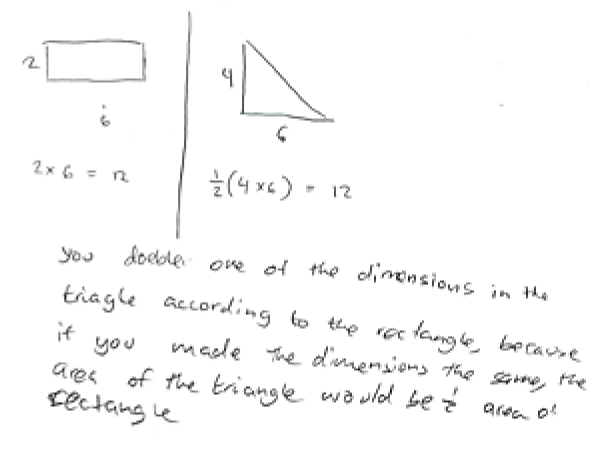
- Below is an open-ended question followed by student responses.
- Look at these responses
- Discuss how you would score the responses using the rubric.

Draw and give the dimensions of a rectangle and a triangle that have the same area. Show that the two figures have the same area.

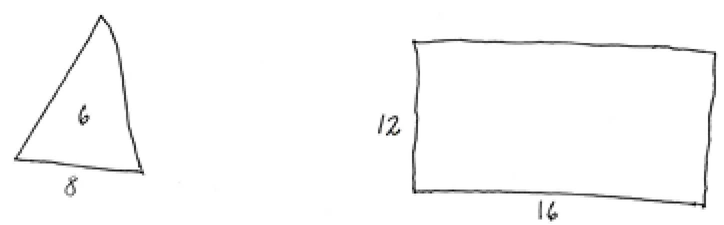
Response A:



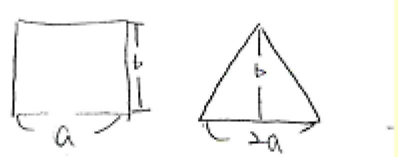
Response B:



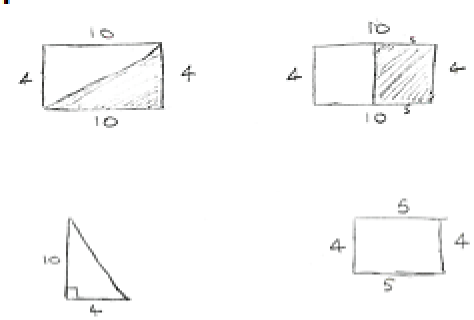
Response C:



Response D:



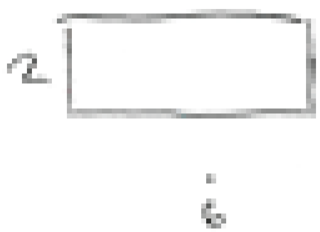
Response E:



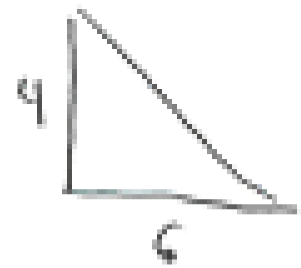
Response A:

Area □ = 32 in²   area △ = 32 in²

Response B:



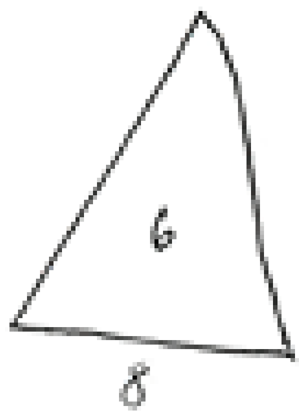
$2 \times 6 = 12$

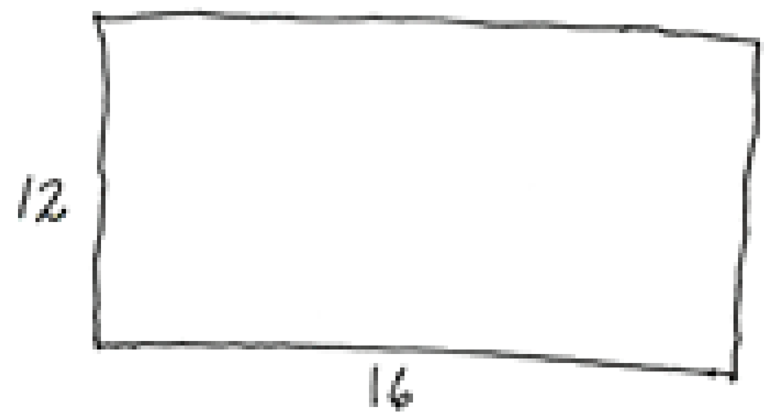


$\frac{1}{2}(4 \times 6) = 12$

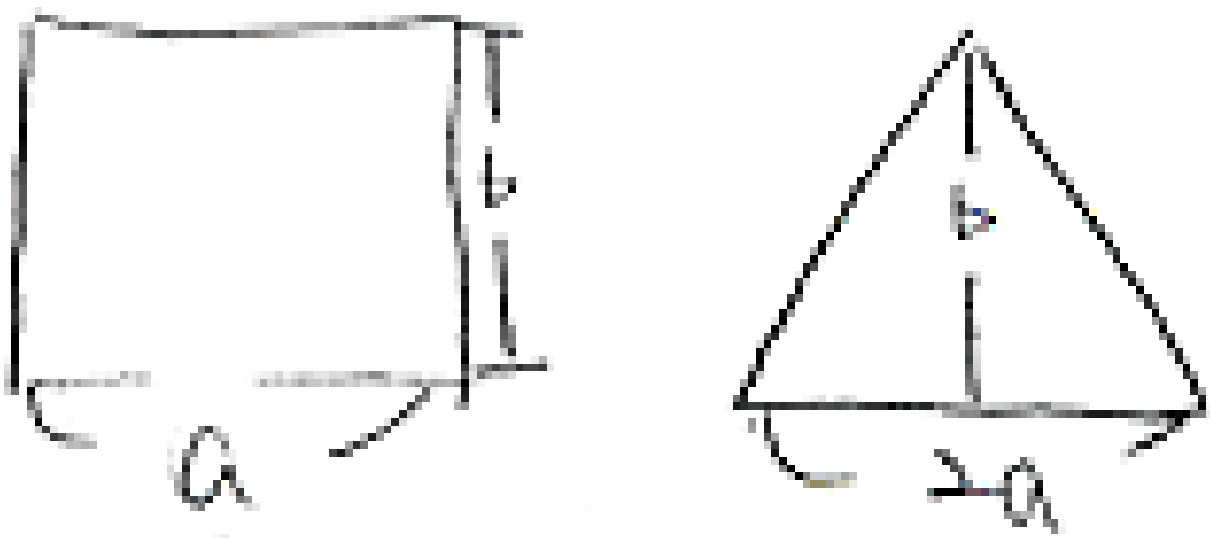
you doddle one of the dimensions in the triagle according to the rectangle, because if you made the dimensions the same, the area of the triangle would be ½ area of rectangle

Response C:

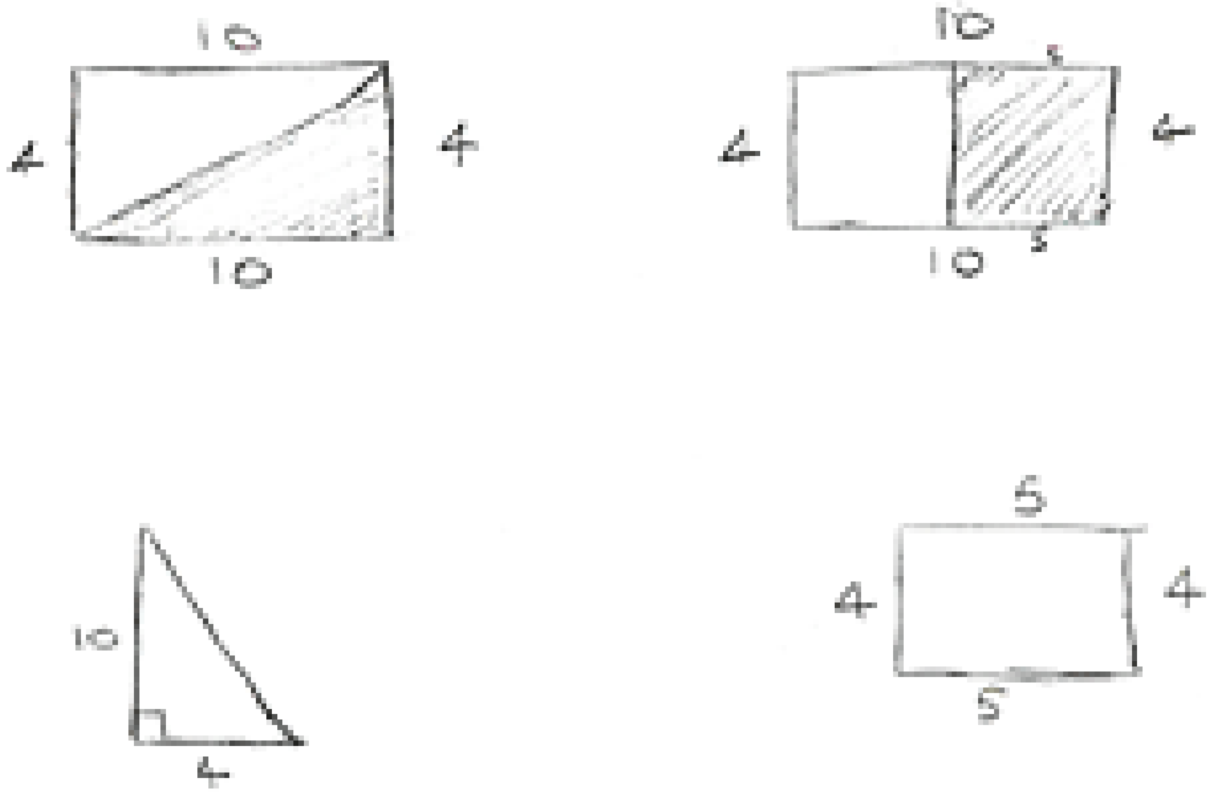




Response D:

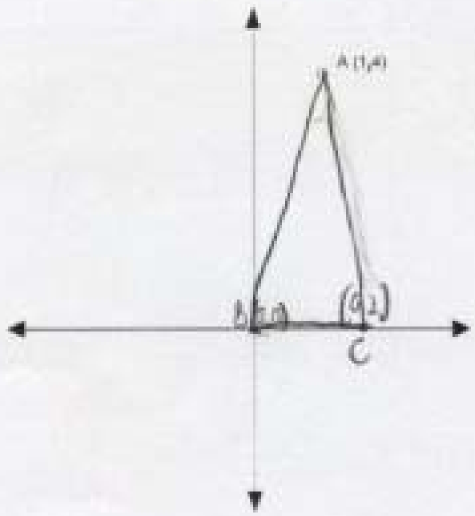


Response E:



- The length of the segment below is the $\sqrt{2}$. Draw a segment that has the length $\sqrt{18}$. Explain how you determine the length of your segment.

- Draw a triangle ABC whose vertices are A(0,0), B(2,0), and C(3,4). Give a convincing argument why you believe the triangle is or is not an isosceles triangle.



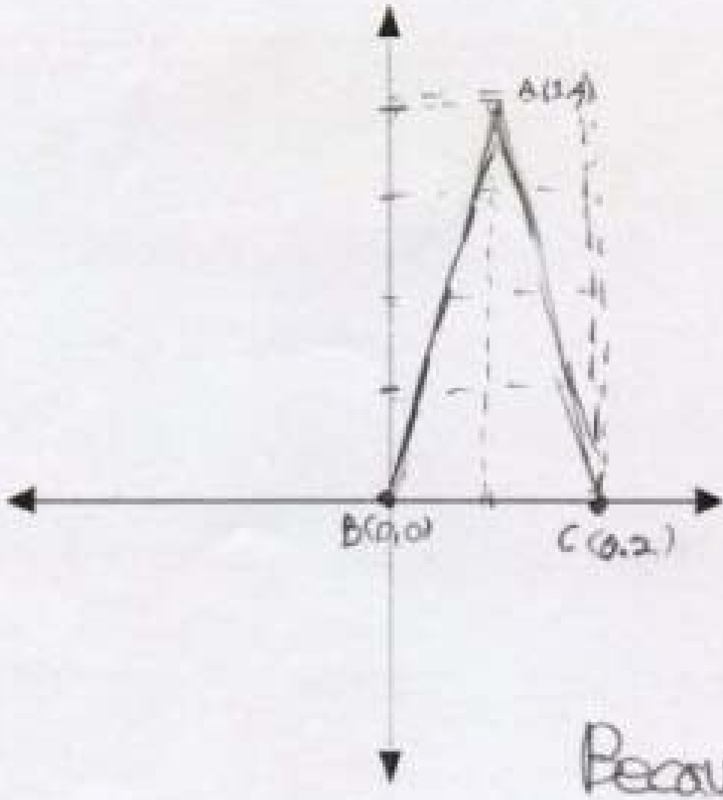
B-0,0

C-0,0

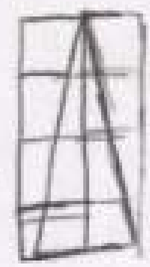
an isosceles triangle has two sides the same length so, we must find the lengths of $\overline{AB}$ and $\overline{AC}$ to prove that this triangle is isosceles

$\overline{AB} = \sqrt{4^2+1^2}$ and $\overline{AC}$ $\sqrt{4^2+1^2}$

so, $\overline{AB}$ and $\overline{AC}$ are the same length, making $\triangle ABC$ isosceles



Because / are.

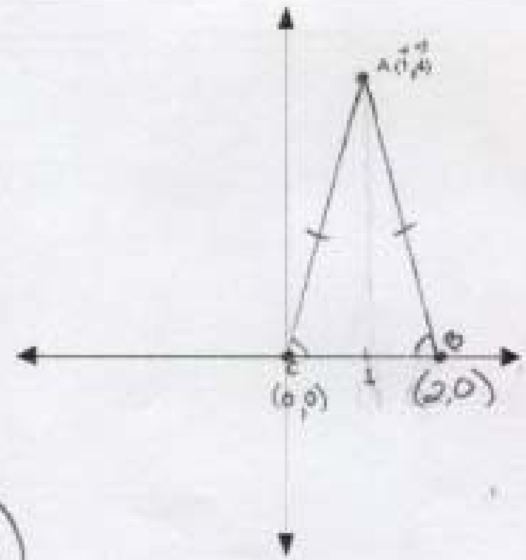


Across 1
Down 4

so it's

same

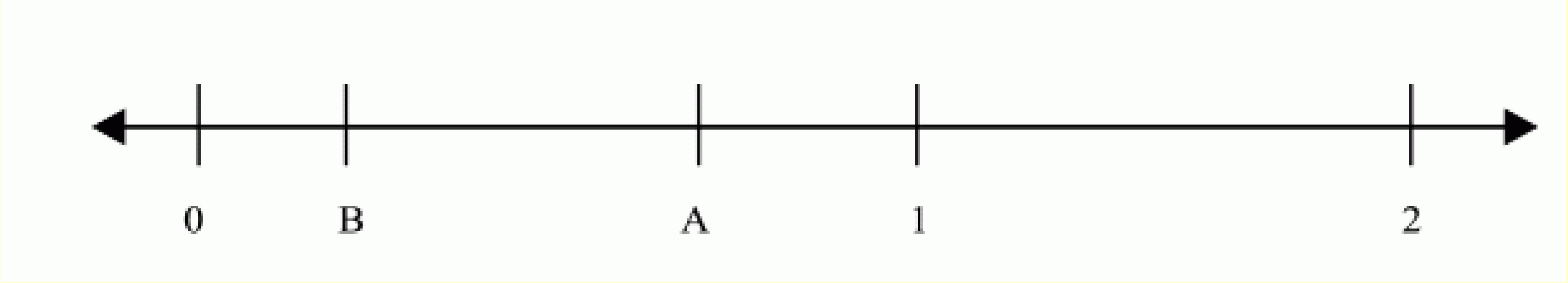
length.

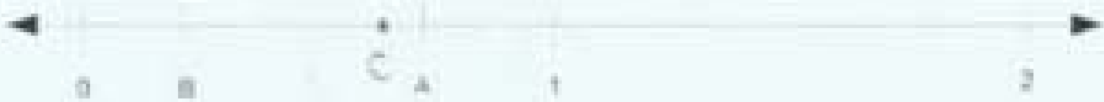


C = (0,0)
B = (2,0)

ΔABC is isosceles because $\overleftrightarrow{AC} = \overleftrightarrow{AB}$.
To have an isosceles triangle, two sides must be equal. So, ΔABC is an isosceles triangle since $\overleftrightarrow{AC}$ and $\overleftrightarrow{AB}$ are equa

- Mark a point representing the number C so that $A \bullet C = B$. Explain how you determined C.





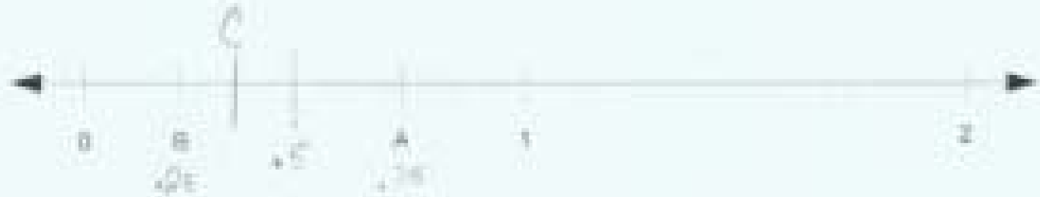
Mark a point representing the number C so that $A \cdot C = B$. Explain how you determined C.

B < A < 1 - A, B must be decimal numbers

C should be less than 1, because if it isn't, then A·C would be greater than B. (A > B)

For ex., if B = 0.3 and A = 0.7 and let's say C is 0. ... something, then 0.7 × 0. ... would be lesser number. If A × something greater than 1, the answer would be larger instead of smaller, which is what we want for B.

So C definitely has to be < 1.



Mark a point representing the number C so that $A \cdot C = B$. Explain how you determined C.

$A \cdot C = B \rightarrow C = \frac{B}{A}$

$\frac{B}{A} = .\overline{3}\ (\frac{1}{3})$

$.\overline{3} = C$

$A \times C$ equals B so C equals B divided by A. So you know $B = .25$ + $A = .75$ so $\frac{.25}{.75} = .\overline{3}$ so $C = .\overline{3}$.

Kind of like fill in the blanks

$\underline{A} \cdot \underline{C} = \underline{B}$ so $\underline{B} \div \underline{A} = \underline{C}$

Mark a point representing the number C so that $A \cdot C = B$. Explain how you determined C.

$\cancel{.75 \times .25}$

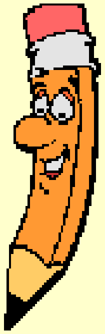
$$\begin{array}{r} \overset{2}{.75} \\ \times\ .50 \\ \hline .3750 \end{array}$$

I multiply A times where I think C is

A is about ~~#~~ .65 and B is about .25

$.65 \div .25 = C = 2.5$

- Talking with your colleagues about student responses to open-ended questions and how to score the responses using the rubric is very helpful.
- When you see that other teachers value many of the same criteria you do, you will feel more comfortable using the rubric.



Writing and Scoring Open-Ended Question in Math

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